Noise of Kondo dot with ac gate: Floquet–Green's function and noncrossing approximation approach

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The transport properties of an ac-driven quantum dot in the Kondo regime are studied by the Floquet-Green's function method with the slave-boson infinite-*U* noncrossing approximation. Our results show that the Kondo peak of the local density of states is robust against a weak ac gate modulation. Significant suppression of the Kondo peak can be observed when the ac gate field becomes strong. The photon-assisted noise of the Kondo resonance as a function of the dc bias does not show singularities which are expected for a noninteracting resonant quantum dot. These findings suggest that one may make use of the photon-assisted noise measurement to tell apart whether the resonant transport is via a noninteracting resonance or a strongly correlated Kondo resonance.

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I. INTRODUCTION

The Kondo effect¹ is a paradigm of the strong correlation in the condensed-matter physics. This effect was discovered in metals with magnetic impurities where the resistivity enhances with decreasing the temperature below a characteristic temperature known as Kondo temperature T_K . This anomaly behavior was theoretically explained by J. Kondo, hence the name "Kondo effect," as the exchange interaction of the itinerant electrons with a localized spin state.¹ Soon, it was realized that the same correlation can dominate the lowtemperature transport properties of a quantum dot with the strong Coulomb interaction.^{2,3} Contrary to the behavior in metals, the Kondo effect leads to an enhancement of the conductivity of the quantum dot. The advance in nanotechnology has experimentally demonstrated the Kondo effect in artificial impurity systems (semiconductor quantum dots,^{4,5} carbon nanotube,⁶ or molecular conductors^{7,8}). The observation of the Kondo effect in these artificial quantum impurities has provided us the opportunity to study correlation phenomena by tuning the relevant parameters or driving the system out of equilibrium by either static or alternate magnetic or electric fields. However, comparing with the great experimental achievements, the theoretical understanding of the Kondo effect, especially at nonequilibrium, is far from adequate. This difficulty is due to the interplay of the strong Coulomb interaction and the nonequilibrium which makes the exact solution of the Kondo problem in nonequilibrium impossible. For the theoretical studies of the Kondo problem, several techniques, such as the equation of motion of the Green's function,⁹ Fermi-liquid theory,^{10,11} the bosonization technique,¹²⁻¹⁴ and the numerical renormalization-group method,¹⁵ with their own advantages and limitations are developed. Most of these studies focus on the spectral density, the linear, or differential conductance and the current.

Nowadays, there is increasing interest in the fluctuation of the current, i.e., the current noise,¹⁶ in the Kondo regime. The reasons for these studies are roughly twofolds. On one hand, the current noise measurement is expected to expose

information about the quantum nature of the electron transport in the Kondo regime. For a Kondo dot, the Kondo correlation is quite sensitive to the external electric or magnetic field. It is thus difficult to get some key information such as the Kondo temperature or the spectral density by traditional experimental measurements. New setups or novel characteristic tools are needed to achieve the desired information. The current noise of a Kondo dot has shown its potential to provide information which is beyond the reach of traditional transport measurements. For instance, Meir and Golub¹⁷ studied the shot noise of a Kondo dot by several complementary approaches. They proposed to estimate T_K by measuring the noise behavior of the Kondo dot. In addition, Sindel et al.¹⁸ pointed out that the current noise can give an estimate of the Kondo resonance in the equilibrium spectral density. On the other hand, as a signature of the many-particle correlation, the current noise in the Kondo regime behaves quite differently from that of systems in the single-particle picture. The recent study of the fractional charge is such an example. It is suggested that the noise measurement in the Kondo regime can lead to noninteger effective charge.¹⁹⁻²¹ Moreover, a recent experiment reveals surprisingly high current noise of a carbon nanotube in the Kondo regime where the conductance is close to the unitary conductance $2e^2/h^{22}$ This is in great contrast to the predictions based on the noninteracting picture where the shot noise is expected to be negligible.

In this study, we show another example where the noise behavior of an ac-driven quantum dot in the Kondo regime differs drastically from that of a noninteracting dot. For noninteracting quantum conductors, Lesovik and Levitov²³ (LL) have predicted that the noise should show singularities at integers of $eV/\hbar\Omega$, where V is the dc bias and Ω is the ac frequency. Soon, this prediction was experimentally verified by measuring the noise of a tunnel junction driven by the microwave field.²⁴ Further theoretical studies, which in principle are still in the single-particle picture, showed that the singularity is robust against the disorder and remains distinct at weak ac field²⁵ (actually, the singularity also depends on the ratio of $eV_{\rm ac}/\hbar\Omega$, where $V_{\rm ac}$ is the ac strength). None of these studies included the correlation effect due to the Coulomb interaction in the conductor. When there is finite Coulomb interaction in the conductor, the noise behavior is expected to show different behaviors since the electrons are correlated and the single-particle picture is no longer valid. Guigou et al.²⁶ studied the current noise in an ac-driven Luttinger liquid. Their results showed that the sharp feature in dS/dV is smoothed by the presence of the Coulomb interaction. Since the electron transport in the Kondo regime is dominated by the strongly correlated Kondo resonance, an immediate question one might ask is what happens to the photon-assisted noise singularities in the ac-Kondo regime. The transport properties of a quantum dot in the ac-Kondo regime have previously been studied.^{27–36} It is found that the Kondo resonance as well the differential conductance will evolve with the ac field in a nontrivial manner. A strong ac field can suppress the Kondo peak either by photon-assisted ionization^{32,36} or decoherence.^{30,31} Interestingly, several studies^{31,33,35} also indicate that the Kondo resonance is robust against an ac gate voltage even when the energy scale of the ac parameters is comparable to the Kondo temperature. Comparing with the works on the conductance or density of states of an ac-driven Kondo dot, there have hitherto been few attempts to study its noise properties.^{29,37,38} Most studies^{29,37} on the noise of the quantum dot in the ac-Kondo regime are based on the equation of motion method. However, the equation of motion method is known not adequate to go deep into the Kondo regime. Recently, the noise of a Kondo dot with the ac gate modulation was studied³⁸ by the present authors by combining the Floquet theorem and the slave-boson mean-field approximation. The numerical results show that the photon-assisted noise of a Kondo dot shows no singularity at zero temperature and very low bias voltage. This smooth noise behavior is in great contrast to the predictions in the single-particle picture. This mean-field approximation is valid when the Kondo temperature defines the largest energy scale. Limited by the validity of the slave-boson mean-field approximation, it remains unclear whether such a singularity can reappear in the Kondo regime when the Kondo temperature does not define the largest energy scale.

It is the purpose of the present paper to investigate the photon-assisted noise of a Kondo dot beyond the slave-boson mean-field approximation. Our approach is based on the noncrossing approximation (NCA).^{12,14,39,40} For $U \rightarrow \infty$, where U is the Coulomb interaction strength, the NCA has been shown to give reliable results both in the temperature regimes above and below T_K . The splitting of the Kondo resonance by external dc bias can be captured. The NCA has also been applied to the time-dependent nonequilibrium transport through a Kondo dot.^{32,33} The solution of the time-dependent NCA in the time domain has been proposed and implemented.³⁹ However, the NCA fails to reproduce the Fermi-liquid behavior at very low temperature $T \ll T_K$ and gives nonphysical results in the mixed-valence region.^{12,41} A suitable remedy of this shortcoming is possible by including higher order self-energies as shown by Kroha *et al.*^{41,42} In the present study, we avoid to go to the pathological regime of the NCA.¹² To make use of the periodicity of the ac field, we combine the Floquet theorem⁴³⁻⁴⁵ and the infinite-U NCA to investigate the transport properties, in particular, the current noise properties of a Kondo dot with ac gate modulation. The Floquet theorem can capture the multiphoton processes in a coherent nonperturbative way. A suitable Floquet-Green's function formalism is developed to describe the nonequilibrium dynamics of the system. Our results show that for the low ac frequency and low ac strength, the local density of states (LDOS) of the Kondo dot is almost insensitive to the ac gate field. For the high ac frequency and strength, the Kondo peak can be significantly suppressed with sidebands appearing in the density of states with distances to the Kondo peak of multiphoton energy. Such suppression is attributed to the decoherence induced by the photon-assisted processes.³⁰ Interestingly, the noise of quantum dots in the ac-Kondo regime shows dramatically different behavior from the acdriven noninteracting dots. The noise as a function of the dc bias voltage does not show singularities in the Kondo regime as we change the ac parameters from weak to strong values. One may thus utilize the photon-assisted noise behavior to tell apart the single-particle noninteracting resonant transport from the strongly correlated Kondo resonance.

The present paper is organized as following. In Sec. II, we give the details of the model Hamiltonian and the Floquet–Green's function formalism with the NCA. In Sec. III, numerical results of the LDOS and the noise properties of an ac-driven Kondo dot are presented. For the sake of comparison, results for a noninteracting dot are given to show the drastic different noise behaviors with or without many-body correlations. In Sec. IV a conclusion is presented.

II. THEORETICAL FORMALISM

A. Model Hamiltonian

The quantum transport through a quantum dot is widely described by the single impurity Anderson model coupled to two ideal leads. The system can be driven out of equilibrium by a dc bias and an ac field. Here, the ac field can be realized via a nearby ac gate which modulates the dot energy level. To investigate the Coulomb effect on the noise behavior, we are interested in two situations, i.e., the noninteracting dot without the Coulomb interaction and the Kondo dot with the strong on-site Coulomb interaction.

For a noninteracting dot, the system can be modeled by the single impurity Anderson model with zero Coulomb strength (U=0) coupled to two ideal leads. The Hamiltonian can be written as

$$H = \sum_{\alpha,k,\sigma} \epsilon_{k,\alpha,\sigma} c_{k,\alpha,\sigma}^{\dagger} c_{k,\alpha,\sigma} + \sum_{k,\alpha,\sigma} (V_{k\alpha\sigma} c_{k,\alpha,\sigma}^{\dagger} d_{\sigma} + \text{h.c.}) + \sum_{\sigma} \epsilon_{d}(t) d_{\sigma}^{\dagger} d_{\sigma}, \qquad (1)$$

where $c_{k,\alpha,\sigma}^{\dagger}$ is the creation operator of a k electron with spin σ in the α th lead ($\alpha=L,R$) and $\epsilon_{k,\alpha,\sigma}$ is the lead electron energy. We assume a symmetric voltage drop across the dot. $V_{k,\alpha,\sigma}$ denotes the coupling matrix element of the lead electron and that in the dot. d_{α}^{\dagger} is the creation operator of a electron in the dot with spin σ . Its energy level can be modulated by an external gate voltage as $\epsilon_d(t) = \epsilon_d^0 + eV_g \cos(\Omega t)$. Here the spin degeneracy is implied.

Usually, the simple assumption of zero Coulomb interaction cannot correctly capture the electron dynamics in nanostructures. For a quantum dot with small size, the charging energy can be much larger than the other energy scales. In these situations, we can approximately take the limit $U \rightarrow \infty$. For infinite U, the dot can contain either zero or one electron. Double occupancy can be neglected. A reliable tool to describe such a quantum dot with its energy level deep below the Fermi energy is the slave-boson NCA.^{13,14,40} The slaveboson technique introduces some auxiliary particles to eliminate the nonquadratic term in the Hamiltonian by replacing the electron operators as $d^{\dagger}_{\sigma} \rightarrow b^{\dagger}f_{\sigma}$ and $d_{\sigma} \rightarrow f^{\dagger}_{\sigma}b_{\sigma}$, where bannihilates the empty state and f_{σ} destroys a spin σ single occupation state. The infinite U Hamiltonian in slave-boson language reads

$$H = \sum_{\alpha k\sigma} \epsilon_{k\alpha\sigma}(t) c^{\dagger}_{k\alpha\sigma} c_{k\alpha\sigma} + \sum_{k\alpha\sigma} (V_{k\alpha\sigma} c^{\dagger}_{k\alpha\sigma} b^{\dagger} f_{\sigma} + \text{H.c.}) + \sum_{\sigma} \epsilon_{d}(t) f^{\dagger}_{\sigma} f_{\sigma}.$$
 (2)

Since the double occupancy is forbidden for $U \rightarrow \infty$, these slave particle operators must satisfy the constraint condition

$$\sum_{\sigma} f_{\sigma}^{\dagger} f_{\sigma} + b^{\dagger} b = 1.$$
(3)

B. Floquet–Green's function formalism

To study the transport properties out of the equilibrium, we make use of the nonequilibrium Green's function method. The physical information is usually stored in two propagators, i.e., the retarded, G', and the lesser, $G^{<}$, Green's functions which are defined in the real time axis. These Green's functions can be related to the contour-ordered Green's function $G_C(t,t')=-i\hbar^{-1}\langle T_C\{\psi(t)\psi^{\dagger}(t')\}\rangle$, along the Keldysh contour, by the Langreth rules.⁴⁶ The retarded Green's function can be found from the Dyson equation in the time domain as

$$\left[i\hbar\frac{\partial}{\partial t} - \epsilon_d(t)\right]G^r(t,t') = \hbar\,\delta(t-t') + \int\,dt''\Sigma^r(t,t'')G^r(t'',t').$$
(4)

The lesser Green's function is given by the Keldysh equation

$$G^{<}(t,t') = \int dt_1 dt_2 G^r(t,t_1) \Sigma^{<}(t_1,t_2) G^a(t_2,t'), \qquad (5)$$

where $G^a(t,t') = [G^r(t',t)]^{\dagger}$ is the advanced Green's function and $\Sigma^{r/<}(t,t')$ is the double-time retarded/lesser self-energy. Usually, we need another Green's function, i.e., the greater Green's function $G^>(t,t')$ which can be found by either using the Keldysh equation by replacing $\Sigma^<$ with the greater self-energy $\Sigma^>$ or from the identity,

$$G^{>}(t,t') - G^{<}(t,t') = G^{r}(t,t') - G^{a}(t,t').$$

For general time-dependent Hamiltonian, the solution of the above equations remains challenging. However, since our Hamiltonian is periodic in time, the Floquet theorem can be used to simplify the solution.^{47,48} It is more desirable to work in an enlarged Hilbert space which is defined by the direct product of the local basis in space and the basis for the timeperiodic functions with the frequency Ω . The set of timeperiodic functions in the form of $\exp(-ik\Omega t)$ with $k \in \mathbb{Z}$ gives the so-called Floquet basis to take into account the periodicity in a nonperturbative way. We will combine the Floquet theory and the Green's functions to investigate the timedependent transport properties of the quantum dot systems.

The following notations are defined and will be used in the rest of the paper. The Fourier transform of a double-time function O(t,t') is defined as

$$O(t,\epsilon) = \int dt' e^{i\epsilon(t-t')} O(t,t').$$
(6)

The periodicity of the function O indicates that it can be decomposed into the Floquet basis as

$$O(t, \epsilon) = \sum_{k} O_{k}(\epsilon) e^{-ik\Omega t},$$
(7)

where O_k is the Fourier coefficient of the function $O(t, \epsilon)$. Several authors have studied the time-dependent transport by the Fourier coefficients of the Floquet–Green's functions,^{49–52} where only single label of the Floquet states is needed. However, as we will see later, it is more convenient to write this time-periodic function O(t, t') in a matrix form O in the Floquet space. The matrix element $O_{k,k'}$ are related to the Fourier coefficient by

$$\mathcal{O}_{k,k'}(\boldsymbol{\epsilon}) = O_{k-k'}(\boldsymbol{\epsilon} + k'\hbar\Omega). \tag{8}$$

In the following, all operators expressed in matrix form in Floquet space will be denoted by calligraphic symbols.

To solve the matrix form Floquet–Green's functions, it is more convenient to work with the Floquet-Hamiltonian which is defined by

$$\mathcal{H} = H(t) - i\hbar \frac{\partial}{\partial t}.$$
(9)

Using the previously introduced notations, the double-time retarded Green's function Eq. (4) can be rewritten in the Floquet space in a compact form as the resolvent of the Floquet-Hamiltonian⁵³ as

$$\mathcal{G}^{r}(\boldsymbol{\epsilon}) = \frac{1}{\boldsymbol{\epsilon} - \mathcal{H}_{dot}(t) - \mathcal{E}^{r}(\boldsymbol{\epsilon})},$$
(10)

where $\mathcal{E}^{r}(\epsilon)$ is the retarded self-energy in the Floquet space and \mathcal{H}_{dot} is the Floquet-Hamiltonian of the isolated dot. The derivation of the above equation is given in the Appendix. Similarly, the advanced Floquet–Green's function is given by

$$\mathcal{G}^{a}(\boldsymbol{\epsilon}) = [\mathcal{G}^{r}(\boldsymbol{\epsilon})]^{\dagger}. \tag{11}$$

After the Fourier transform and rewriting in the matrix form, the lesser Floquet–Green's function which is given by Eq. (5) in the time domain can be rewritten in a compact form as

$$\mathcal{G}^{<}(\boldsymbol{\epsilon}) = \mathcal{G}^{r}(\boldsymbol{\epsilon})\mathcal{E}^{<}(\boldsymbol{\epsilon})\mathcal{G}^{a}(\boldsymbol{\epsilon}), \qquad (12)$$

where $\mathcal{E}^{<}$ represents the lesser self-energy in Floquet space.

We can see that with the help of the Floquet theorem and written the quantities in the matrix form, both the retarded and lesser Green's functions [Eqs. (10) and (12)] have similar structures with the Dyson and Keldysh equations for the Green's function in stationary situations,⁵⁴ though these Green's functions in the time domain are double-time functions. The main difference is that the Floquet–Green's function is expressed in an enlarged Hilbert space due to the periodicity condition.

C. Noninteracting quantum dot

For a noninteracting dot, i.e., U=0, the Hamiltonian of the system can be given with quadratic terms only. We assume the energy level of the dot is modulated by a harmonic field. The dot Hamiltonian takes the time dependence $H_{dot}(t)=\sum_{\sigma}[\epsilon_d+eV_g\cos(\Omega t)]d_{\sigma}^{\dagger}d_{\sigma}$. The corresponded Floquet-Hamiltonian is then given in matrix form by

$$[\mathcal{H}_{dot}]_{k,k'} = \{\epsilon_d + k\hbar\Omega\}\delta_{k,k'} + \frac{1}{2}eV_g\delta_{k,k'\pm 1}.$$
 (13)

In the presence of an ac gate, the self-energies are due to the coupling between the dot and the leads. For the dot-lead coupling, the retarded and lesser self-energies are given in the time domain as

$$\Sigma_{\alpha,\sigma}^{r}(t,t') = \sum_{k} V_{k\alpha}^{2} g_{k\alpha\sigma}^{r}(t,t'), \qquad (14)$$

$$\Sigma_{\alpha,\sigma}^{<}(t,t') = \sum_{k} V_{k\alpha}^{2} g_{k\alpha\sigma}^{<}(t,t'), \qquad (15)$$

where g represents the Green's function of the lead electrons at equilibrium and α is the lead label. The retarded and lesser Green's functions of the lead electrons are given, respectively, by

$$g_{k\alpha\sigma}^{r}(t,t') = -i\theta(t-t')\exp\left[-i\int_{t'}^{t} dt_{1}\epsilon_{k\alpha}(t_{1})\right], \quad (16)$$

$$g_{k\alpha\sigma}^{<}(t,t') = if_{\alpha}(\boldsymbol{\epsilon}_{k\alpha}^{0}) \exp\left[-i \int_{t'}^{t} dt_{1} \boldsymbol{\epsilon}_{k\alpha}(t_{1})\right], \quad (17)$$

where $f_{\alpha}(\boldsymbol{\epsilon})$ is the Fermi-distribution function in the α lead.

After the Fourier transform and some algebra, the selfenergies [Eq. (14)] can be rewritten in the Floquet basis as

$$\mathcal{E}_{\alpha,\sigma;k,k'}^{r/<}(\epsilon) = \Sigma_{\alpha\sigma}^{r/<}(\epsilon + k'\hbar\Omega)\delta_{k,k'}.$$
 (18)

Once we neglect the energy shift by the dot-lead coupling, the static self-energy Σ_{α} can be given in the so-called wideband approximation as

$$\Sigma_{\alpha}^{r}(\omega) = -\frac{i}{2}\Gamma_{\alpha}(\omega), \qquad (19)$$

$$\Sigma_{\alpha}^{<}(\omega) = i\Gamma_{\alpha}(\omega)f_{\alpha}(\omega), \qquad (20)$$

where $\Gamma_{\alpha}(\omega) = 2\pi \Sigma_{k \in \alpha} V_k^2 \delta(\omega - \epsilon_k) = 2\pi \rho_{\alpha}(\omega) V^2$ and ρ_{α} is the density of states of the α lead.

The transport properties can be expressed by the Green's functions and the self-energies of the quantum dot. The expression for the time-dependent current operator from the left lead is related to the time derivation of the total number operator in the left lead as

$$J_L(t) = -e \frac{\partial N_L}{\partial t} = -\frac{ie}{\hbar} [H(t), N_L], \qquad (21)$$

where $N_L = \sum_{k\sigma} c_{k,L,\sigma}^{\dagger} c_{k,L,\sigma}$. In terms of the nonequilibrium Green's functions, the expectation value of the current can be found by

$$\langle J_L(t) \rangle = \frac{2e}{\hbar} \sum_{\sigma} \lim_{t' \to t} \operatorname{Re} \left\{ \int dt_1 [G_{\sigma}^r(t,t_1) \Sigma_{L,\sigma}^<(t_1,t') + G^<(t,t_1) \Sigma_L^a(t_1,t')] \right\},$$
(22)

where Σ_L^a is the advanced self-energy due to the coupling to the left lead.

Due to the discrete nature of electrons, the current fluctuation is nonzero even at zero temperature. This fluctuation, known as current noise, contains information of electron correlations which is beyond the reach of traditional conductance measurement. This current noise may be defined as¹⁶

$$S_{\rm LL}(t,t') = \frac{1}{2} \langle \{ \Delta J_L(t), \Delta J_L(t') \} \rangle, \qquad (23)$$

where $\Delta J_L(t) = J_L(t) - \langle J_L(t) \rangle$ represents the fluctuation of the left lead current operator from its expectation value.

Using the Floquet–Green's functions in the matrix form, the time-dependent current can be rewritten in a compact form which shares a similar structure of the timeindependent Meir-Wingreen current formula as

$$\langle J_L(t)\rangle = \frac{4e}{\hbar} \operatorname{Re}\left\{\int \frac{d\omega}{2\pi} [\mathcal{G}^r \mathcal{E}_L^< + \mathcal{G}^< \mathcal{E}_L^a]_{k,0}(\omega) e^{-ik\Omega t}\right\},\tag{24}$$

where \mathcal{E}_L^a and $\mathcal{E}_L^<$ are the advanced and lesser self-energies in the Floquet space due to the coupling to the left lead. To find the time-averaged current over one period, one can simply set k=0 in the above formula. For time-dependent transport, there is displacement current due to charge accumulation in the dot. However, the contribution of the displacement current to the time-averaged current vanishes.⁵⁵

Inserting the current operator in the noise definition and after some algebra,⁵⁶ the *time-averaged* current fluctuation at zero frequency can be given in the matrix form of the Floquet–Green's functions and self-energies as

$$\hat{S}_{LL}(\omega=0) = \frac{2e^2}{h} \left[\int d\epsilon \mathcal{E}_L^{>} \mathcal{G}^{<} + \mathcal{G}^{>} \mathcal{E}_L^{<} - (\mathcal{G}^r \mathcal{E}_L^{>} + \mathcal{G}^{>} \mathcal{E}_L^{a}) \right]$$

$$\times (\mathcal{G}^r \mathcal{E}_L^{<} + \mathcal{G}^{<} \mathcal{E}_L^{a}) + \mathcal{G}^{>} (\mathcal{E}_L^r \mathcal{G}^r \mathcal{E}_L^{<} + \mathcal{E}_L^r \mathcal{G}^{<} \mathcal{E}_L^{a} + \mathcal{E}_L^{<} \mathcal{G}^a \mathcal{E}_L^{a})$$

$$+ (\mathcal{E}_L^r \mathcal{G}^r \mathcal{E}_L^{>} + \mathcal{E}_L^r \mathcal{G}^{>} \mathcal{E}_L^{a} + \mathcal{E}_L^{>} \mathcal{G}^a \mathcal{E}_L^{a}) \mathcal{G}^{<} - (\mathcal{E}_L^r \mathcal{G}^{>} + \mathcal{E}_L^{>} \mathcal{G}^a)$$

$$\times (\mathcal{E}_L^r \mathcal{G}^{<} + \mathcal{E}_L^{<} \mathcal{G}^a) + \text{H.c.} \right]_{0,0}.$$
(25)

The above noise formula is exact for a noninteracting dot where the Hamiltonian of the system is quadratic so that the Wick theorem can be applied. In the presence of finite Coulomb interaction, we have quartic terms. A direct application of the Wick theorem is no longer possible. A full diagrammatic expansion is needed to reach the desired formula. However, such task remains formidable since multiparticle Green's functions are required. In order to make use of the power of the Wick theorem, one has to rely on approximations such as the mean-field approximation or slave-boson techniques to rewrite the Hamiltonian in quadratic forms.

D. Infinite-U Kondo dot: Slave-boson NCA

For infinite U, a well-established tool to study the transport properties in the Kondo regime is the NCA.^{12,14,39,40} The NCA is the lowest-order conserving approximation. It is widely used to investigate the properties of a Kondo dot at equilibrium or out of equilibrium. Using the NCA, the self-energies of the pseudoparticles are given in the time domain as^{12,14,39,40}

$$\Xi^{r}_{\sigma}(t,t') = i V^{2} \sum_{k\alpha} g^{>}_{k\alpha\sigma}(t,t') B^{r}(t,t'), \qquad (26)$$

$$\Pi^{r}(t,t') = -iV^{2}\sum_{k\alpha\sigma}D^{r}(t,t')g^{<}_{k\alpha\sigma}(t',t), \qquad (27)$$

$$\Xi_{\sigma}^{<}(t,t') = iV^{2}\sum_{k\alpha\sigma} g_{k\alpha\sigma}^{<}(t,t')B^{<}(t,t'), \qquad (28)$$

$$\Pi^{<}(t,t') = -iV^2 \sum_{k\alpha\sigma} D^{<}(t,t') g^{>}_{k\alpha\sigma}(t',t), \qquad (29)$$

where $g^>$ is the greater Green's function of electrons in leads,

$$g_{k\alpha\sigma}^{>}(t,t') = -i[1 - f_{\alpha}(\epsilon_{k\alpha\sigma}^{0})]\exp\left[-i\int_{t'}^{t} dt_{1}\epsilon_{k\alpha\sigma}(t_{1})\right].$$
(30)

 Ξ is the self-energy of the pseudofermion while Π represents the self-energy for the pseudoboson Green's functions.

The double-time Green's functions for the slave particles are defined by

$$iD_{\sigma}(t,t') = \langle T_c f_{\sigma}(t) f_{\sigma}^{\dagger}(t') \rangle, \qquad (31)$$

$$iB(t,t') = \langle T_c b(t) b^{\dagger}(t') \rangle.$$
(32)

The retarded and lesser Green's functions for the pseudoboson and the pseudofermion, together with their selfenergies are obtained from a set of self-consistent equations. The equations for the retarded and lesser Green's functions in the time domain are given by⁴⁰

$$\left(i\frac{\partial}{\partial t} - \epsilon_{\sigma}\right) D_{\sigma}^{r}(t,t') = \delta(t,t') + \int dt_{1} \Xi_{\sigma}^{r}(t,t_{1}) D_{\sigma}^{r}(t_{1},t'),$$

$$(33)$$

$$i\frac{\partial}{\partial t} B^{r}(t,t') = \delta(t,t') + \int dt_{1} \Pi^{r}(t,t_{1}) B^{r}(t_{1},t'), \quad (34)$$

$$\left(i\frac{\partial}{\partial t} - \epsilon_d\right) D_{\sigma}^{<}(t,t') = \int dt_1 [\Xi_{\sigma}^{r}(t,t_1) D_{\sigma}^{<}(t_1,t') + \Xi_{\sigma}^{<}(t,t_1) D^{a}(t_1,t')],$$
(35)

$$i\frac{\partial}{\partial t}B^{<}(t,t') = \int dt_1 [\Pi^r(t,t_1)B^{<}(t_1,t') + \Pi^{<}(t,t_1)B^a(t_1,t')].$$
(36)

The greater and advanced Green's functions are given in the NCA as

$$D_{\sigma}^{>}(t,t') = i[D^{r}(t,t') - D^{a}(t,t')], \qquad (37)$$

$$B^{>}(t,t') = i[B^{r}(t,t') - B^{a}(t,t')], \qquad (38)$$

$$D^{a}(t,t') = [D^{r}(t',t)]^{*}, \qquad (39)$$

$$B^{a}(t,t') = [B^{r}(t',t)]^{*}.$$
(40)

Making use of the periodicity of our problem, the above equations can again be rewritten in the matrix form in an enlarged Hilbert space. The Floquet–Green's functions of the pseudofermion and pseudoboson can then be given in a compact matrix form as

$$\mathcal{D}_{\sigma}^{r}(\boldsymbol{\epsilon}) = \frac{1}{\boldsymbol{\epsilon} - \mathcal{H}_{f} - \mathcal{K}_{\sigma}^{r}(\boldsymbol{\epsilon})},$$
(41)

$$\mathcal{B}^{r}(\boldsymbol{\epsilon}) = \frac{1}{\boldsymbol{\epsilon} - \mathcal{H}_{b} - \mathcal{P}^{r}(\boldsymbol{\epsilon})},\tag{42}$$

$$\mathcal{D}_{\sigma}^{<} = \mathcal{D}_{\sigma}^{r} \mathcal{K}_{\sigma}^{<} \mathcal{D}_{\sigma}^{a}, \tag{43}$$

$$\mathcal{B}^{<} = \mathcal{B}^{r} \mathcal{P}^{<} \mathcal{B}^{a}, \tag{44}$$

where \mathcal{H}_f and \mathcal{H}_b are the Floquet-Hamiltonians of the pseudofermion and the pseudoboson, respectively. $\mathcal{D}(\mathcal{B})$ is the Floquet–Green's function of the pseudofermion (pseudoboson) in the matrix form. Their corresponding self-energies in the matrix form are denoted as \mathcal{K} and \mathcal{P} , respectively. The matrix elements of the self-energies for pseudoparticles within the NCA are given in the Floquet space by

$$\mathcal{K}_{\sigma;k_1,k_2}^r(\omega) = \sum_{k'} \int \frac{d\omega'}{2\pi} \mathcal{E}_{\sigma;k_1,k'}^{>}(\omega' - k'\hbar\Omega) \mathcal{B}_{k',k_2}^r(\omega - \omega'),$$
(45)

$$\mathcal{K}_{\sigma;k_1,k_2}^{<}(\omega) = \sum_{k'} \int \frac{d\omega'}{2\pi} \mathcal{E}_{\sigma;k_1,k'}^{<}(\omega' - k'\hbar\Omega) \mathcal{B}_{k',k_2}^{<}(\omega - \omega'),$$
(46)

$$\mathcal{P}_{k_1,k_2}^r(\omega) = -\sum_{\sigma,k'} \int \frac{d\omega'}{2\pi} \mathcal{D}_{\sigma,k_1,k'}^r(\omega+\omega') \mathcal{E}_{\sigma,k',k_2}^<(\omega'-k_2\hbar\Omega),$$
(47)

$$\mathcal{P}_{k_1,k_2}^{<}(\omega) = -\sum_{\sigma,k'} \int \frac{d\omega'}{2\pi} \mathcal{D}_{\sigma;k_1,k'}^{<}(\omega+\omega') \mathcal{E}_{\sigma;k',k_2}^{>}(\omega'-k_2\hbar\Omega).$$
(48)

These equations can be solved in a self-consistent way. At each iteration in the numerical calculations, the constraint condition (3) is numerically checked as

$$\int \frac{i}{2\pi} d\epsilon \left[\mathcal{B}^{<}(\epsilon) - \sum_{\sigma} \mathcal{D}_{\sigma}^{<}(\epsilon) \right] = 1.$$
(49)

From the time derivation of the total electron number in the left lead, the time-averaged current formula can be found with the help of the Floquet–Green's functions as

$$J = \frac{2e}{\hbar} \sum_{\sigma} \int \frac{d\epsilon}{2\pi} \{ [\mathcal{D}_{\sigma}^{r} \mathcal{K}_{\sigma}^{<} + \mathcal{D}_{\sigma}^{<} \mathcal{K}_{\sigma}^{a}](\omega) \}_{0,0}.$$
(50)

Due to the strong correlations, an exact formula for the noise in the Kondo regime is formidable, if it exists. One has to make some approximations. Along the line for the derivation of Eq. (25), a noise expression using the slave particle Floquet–Green's functions within the NCA can be obtained as

$$\begin{split} \hat{S}_{LL}(\omega=0) &= \frac{2e^2}{h} \Biggl[\int d\epsilon \mathcal{K}_L^{>} \mathcal{D}^{<} + \mathcal{D}^{>} \mathcal{K}_L^{<} \\ &- (\mathcal{D}^r \mathcal{K}_L^{>} + \mathcal{D}^{>} \mathcal{K}_L^a) (\mathcal{D}^r \mathcal{K}_L^{<} + \mathcal{D}^{<} \mathcal{K}_L^a) \\ &+ \mathcal{D}^{>} (\mathcal{K}_L^r \mathcal{D}^r \mathcal{K}_L^{<} + \mathcal{K}_L^r \mathcal{D}^{<} \mathcal{K}_L^a + \mathcal{K}_L^{<} \mathcal{D}^a \mathcal{K}_L^a) \\ &+ (\mathcal{K}_L^r \mathcal{D}^r \mathcal{K}_L^{>} + \mathcal{K}_L^r \mathcal{D}^{>} \mathcal{K}_L^a + \mathcal{K}_L^{>} \mathcal{D}^a \mathcal{K}_L^a) \mathcal{D}^{<} \\ &- (\mathcal{K}_L^r \mathcal{D}^{>} + \mathcal{K}_L^{>} \mathcal{D}^a) (\mathcal{K}_L^r \mathcal{D}^{<} + \mathcal{K}_L^{<} \mathcal{D}^a) + \text{H.c.} \Biggr]_{0,0}. \end{split}$$

$$(51)$$

In the above derivations, we have, following Meir and Golub,¹⁷ neglected the vertex correction when decoupling the correlation functions. One merit of this approximation is that it can hold the fluctuation-dissipation theorem at zerobias voltage.¹⁷ This merit has been verified in our numerical results. In the following, we will use the above noise formula to investigate the noise properties of an ac-driven Kondo dot.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we apply the Floquet–Green's function approach with the infinite-U NCA to calculate the LDOS and noise properties of an ac-driven Kondo dot. In the following calculations, we take the Lorentzian shape LDOS (Ref. 14) of the lead. The coupling between the dot and the lead is assumed to be

$$\Gamma_{\alpha}(\omega) = \Gamma_0 \frac{D^2}{(\omega - \mu_{\alpha})^2 + D^2},$$
(52)

where D is the half bandwidth of the lead and μ_{α} is the chemical potential. The finite bandwidth is introduced to prevent the ultraviolet divergency in the numerical calculation.



FIG. 1. (Color online) LDOS of a Kondo quantum dot in the presence of the ac gate modulation for different ac parameters. The ac strength is fixed at $eV_{\rm ac}/\hbar\Omega=1$. The inset displays the evolution of the Kondo peak with increasing ac field. For low values of the ac parameters ($\Omega < 20T_K$), the changes in the Kondo peak are nearly undistinguishable.

We will take $\Gamma_0=1$ as the energy unit in the following discussions. For the sake of simplicity, we use $\hbar = e = k_B = 1$. Throughout the calculations, we have fixed $\epsilon_d^0 = -2$ and D=25 without other statement. The Kondo temperature for infinite U can be estimated from $T_K = \frac{D}{\sqrt{2\pi}|\epsilon_d^0|} \exp(-\pi|\epsilon_d^0|) \approx 0.013$. The temperature is chosen as T=0.01 without other statement which is below the estimated Kondo temperature and not in the pathology regime of the NCA.

A. Time-averaged LDOS of Kondo dot

One fingerprint of the Kondo effect is the sharp Kondo peak of the LDOS at the Fermi energy. In the following, we study the time-averaged LDOS of the Kondo dot which is driven by an ac gate voltage. The LDOS of the Kondo dot is given by the retarded Green's function of the dot electrons which can be found from

$$G_{\sigma}^{r}(t,t') = i[D_{\sigma}^{r}(t,t')B^{<}(t',t) + D_{\sigma}^{<}(t,t')B^{a}(t',t)].$$
(53)

At equilibrium, the retarded Green's function after the Fourier transform is independent of the time. When the system is driving by an ac field, the retarded Green's function after the Fourier transform is time-dependent and can be given in the Floquet space as

$$[\mathcal{G}_{\sigma}^{r}(\omega)]_{k_{1},k_{2}} = i \int \frac{d\omega'}{2\pi} [\mathcal{D}_{\sigma;k_{1},k'}^{r}(\omega+\omega')\mathcal{B}_{k',k_{2}}^{<}(\omega'-k_{2}\hbar\Omega) + \mathcal{D}_{\sigma;k_{1},k'}^{<}(\omega+\omega')\mathcal{B}_{k',k_{2}}^{a}(\omega'-k_{2}\hbar\Omega)].$$
(54)

The time-averaged LDOS can then be easily obtained by

$$\rho(\omega) = -\frac{1}{\pi} \text{Im}[\mathcal{G}^r(\omega)]_{0,0}.$$
(55)

In Fig. 1, we show the calculated LDOS $\rho(\omega)$ of the quantum dot as a function of the energy ω . The dot is modulated by an ac gate via $\epsilon_d(t) = \epsilon_d^0 + eV_{\rm ac}\cos(\Omega t)$. Different ac parameters are used in our calculated as indicated in the figure. The ratio between the ac strength and the ac frequency is fixed at $\Omega/V_{\rm ac}=1$. From Fig. 1, we can see that the ac gate

voltage can modify the time-averaged LDOS. However, the Kondo peak is not suppressed monotonically by increasing the ac frequency (ac strength). The evolution of the Kondo peak with increasing ac field is enlarged in the inset of Fig. 1. For the low ac field (ac frequency and ac strength), for example, $\Omega = 1T_{K}$, the time-averaged LDOS is almost identical with the equilibrium LDOS. The LDOS against the ac field remains robust in the numerical results as up to an external ac frequency of $20T_K$ as shown in the inset of Fig. 1. This value is much larger than the width of the Kondo peak which can be estimated by its Kondo temperature T_K . From Fig. 1, by increasing the ac frequency from $1T_K$ to $20T_K$, one can find that the broad peak around ϵ_d^0 is only slightly lowered while the Kondo peak is almost unchanged. For these low frequencies, the time-averaged LDOS largely resembles the averaged LDOS for quantum dot at equilibrium with timedependent energy level over one period of the ac modulation. The observed robust of the LDOS against not-strong the weak ac field agrees well with the findings reported in Ref. 33 where the Kondo dot is found to be insensitive to a weak ac field. These observations are reasonable because the ac gate voltage only modulates the energy level of the quantum dot. The Kondo peak is not modulated by the ac gate directly. For the weak ac field and in the adiabatic limit, we may approximately deem the Kondo temperature which determines the Kondo resonance peak, changes adiabatically with the modulation of the energy level $\epsilon_d = \epsilon_d^0 + V_{ac} \cos(\Omega t)$ following the estimation of the Kondo temperature as $T_K = \frac{D}{\sqrt{2\pi|\epsilon_d|}} \exp(-\pi|\epsilon_d|)$. Since the energy level of the quantum dot is deep below the Fermi energy and the ac voltage is much smaller than the distance between the energy level and the Fermi energy $V_{\rm ac} \ll \epsilon_d^0$, the ac field has essentially no effect on the Kondo resonance. However, when the ac field is strong enough to be comparable to the coupling strength or the absolute value of the energy level, the adiabatic picture is no longer valid. Significant suppression of the Kondo peak can be induced by the ac field. At the same time, the width of the Kondo peak is broadened. For these very large ac fields $(\Omega = 50T_K \text{ or } 100T_K \text{ in Fig. 1})$, the numerical results clearly show the inelastic photon-assisted processes. One can observe in Fig. 1 that some replicas of the Kondo peak appear in the LDOS with a distance to the Kondo peak of the ac frequency.

B. Photon-assisted noise for quantum dot

For a noninteracting quantum dot, the analysis based on the scattering approach has shown that the photon-assisted noise as a function of the dc bias displays cusps at integer $\Omega/V.^{23}$ The derivation of the noise then gives the staircase behavior. Such singularities can be attributed to the change in the distribution of the transmitted charge by an ac field. Here, we calculate the photon-assisted noise through a noninteracting quantum dot with an ac gate by the Floquet– Green's function method and the noise formula, Eq. (25). Identical numerical results with Eq. (25) are also obtained by the noise formula presented in Ref. 49 which is valid for the ac transport without the Coulomb interaction. In Fig. 2, we present the time-averaged noise power (in unit $\frac{e^2\Gamma_0}{h}$) as a



FIG. 2. (Color online) Noise of a noninteracting quantum dot with an ac gate modulation as a function of the applied dc bias for different values of the energy level.

function of the dc bias voltage V for different dot energy levels. In the calculation, the temperature is fixed at T=0.002. The ac frequency is chosen as $\Omega = 0.2$ and the ac strength is $V_{ac} = \Omega$. Since the magnitude of the noise varies a lot with the energy-level positions, we have rescaled the y axis to make the time-averaged noise S(V) behavior at $V/\Omega = 1$ clearer. Figure 2 indicates that the appearance of the cusp in the photon-assisted noise depends on the position of the quantum dot energy level. When $\epsilon_d = 0$, i.e., the dot is in the tunneling resonance, the noise shows an obvious cusp at $V=\Omega$. The cusp is ascribed to the nonadiabatic photonassisted tunneling.²³ Since we have a nonzero temperature, the sharpness of this cusp is smoothed. When we lower the energy level and tune the dot out of the resonance, the noise curves become smooth around $eV_{dc} = \hbar \Omega_{ac}$. These results indicate that the singularity of dS/dV for a noninteracting dot in the presence of an ac gate modulation is most pronounced when the dot is at resonance. However, when the dot is far from resonance, the noise behaviors become smooth and it is hard to experimentally detect the noise singular behavior.

We have shown that the noninteracting resonant dot can display the noise singularity when the dot is modulated by an ac gate. An intuitive analogous between the noninteracting resonant dot and the Kondo resonance, where both resonances can reach the unitary conductance $(2e^2/h \text{ with spin})$ degeneracy), may lead to the naive conclusion that the noise of a Kondo dot will show singularity in the presence of an ac gate since there is the Kondo resonance at the Fermi energy. However, this statement is not correct, at least in the strongly correlated infinite-U regime, as shown in our numerical results. In Fig. 3, we show the noise properties of the Kondo dot with different ac frequencies as a function of the applied dc voltage. Since we are interested in the noise behavior at $V/\Omega=1$, the x axis is scaled by the ac frequency to have a better view at $V/\Omega = 1$ (note the different value of Ω in each curve). In Fig. 3, no singularity is observed at $V/\Omega = 1$ for all the ac parameters. For lowest ac frequency $(\Omega = 1T_K)$, this is in agreement with our previous results obtained by the slaveboson mean-field approximation³⁸ where the noise at weak ac field is almost identical to the numerical results without ac field. It is interesting to see that both the slave-boson meanfield approximation and the NCA, which are valid in their respective parameter spaces, show that the noise of a Kondo dot remains almost unaffected by the ac gate. The Kondo dot



FIG. 3. (Color online) Noise of a Kondo dot as a function of the applied dc bias at fixed $eV_{ac}/\hbar\Omega=1$ and for four different values of the frequency (in unit of the Kondo temperature). No singularities are observed at $eV_{dc}/\hbar\Omega=1$ for the different ac field parameters.

behaves as if the ac gate is effectively screened to induce singularities in the noise behavior. Comparing with the noninteracting resonant tunneling, the disappearance of a singular behavior in the noise of the Kondo dot may be ascribed to the fact that the electrons through the Kondo resonance are not directly modulated by the ac field. Although the energy level of the quantum dot ϵ_d^0 is periodically modulated by the ac gate, the Kondo peak is not directly driven by the ac field. For the weak ac field where the ac frequency or ac strength is not in the order of the coupling strength or the energy level, the ac gate voltage is too small to drive the dot out of the Kondo regime. The strong correlation can still give rise to a sharp Kondo peak at the Fermi energy as shown in Fig. 1. From Fig. 1, we can see that the ac modulation of Kondo peak is robust against the ac modulation. When the electrons are tunneling though the dot via the Kondo resonance, their transport behavior is effectively not significantly modulated by the ac field. The influence of the time periodicity of the Kondo peak to the transport electrons is negligibly small to show any singularities in the noise behavior. However, when the ac strength is strong enough to be comparable to ϵ_d^0 or Γ , for example, $\Omega = 20T_K$ in Figs. 1 and 3, the quantum dot can sometimes be driven out of the Kondo regime by the ac field. This can be intuitively understood in an adiabatic picture. With a large amplitude of the oscillating voltage, the ac gate voltages can sometimes bring the energy level of the quantum dot into the energy regimes where the corresponding Kondo temperature becomes much lower than the temperature. In this way, the magnitude of the Kondo peak can then be significantly suppressed. As a consequence, the conductance declines drastically and deviates from the unitary limit. In these situations, the electron transmission probability becomes too small to show significant noise singularity.

Since the cusp of the noise behavior can be smoothed by the finite temperature, it is important to exclude the possibility that the smooth noise behavior shown in Fig. 3 is due to the finite temperature ($T=0.01 < T_K$) in our numerical simulation. In Fig. 4, we plot the noise behavior as a function of the dc bias voltage for the dot with different temperatures. The ac frequency and ac strength are fixed at $\Omega = V_{ac} = 1T_K$. The temperatures are chosen to be 0.01, 0.005, and 0.001, respectively. From Fig. 4, we can see that even for the lowest temperature (T=0.001) which is much lower than T_K , no



FIG. 4. (Color online) Noise of a Kondo dot as a function of the applied dc bias for different temperatures. The ac frequency and the ac strength are fixed at $1T_K$.

cusp is observed at $V/\Omega=1$ in the noise behavior. Together with the numerical results presented in Ref. 38 where no singularity is observed at zero temperature, the disappearance of the cusp in the noise cannot be ascribed to the finite temperature. It is a generic feature of the strongly correlated Kondo peak.

Usually, both the noninteracting resonant tunneling and the Kondo resonance will lead to the unitary conductance in the I-V measurement. One cannot tell part these two transports by the conductance measurement. Our numerical results together with those of Ref. 38 have clearly shown that the strongly correlated Kondo resonance can display a quite different noise behavior as compared with the noninteracting dot. Therefore, it is possible to make use of the distinct photon-assisted noise behavior to tell apart whether the resonant transport is via the noninteracting resonance or the strongly correlated Kondo resonance. Such measurement has been in the reach of the present technology. The recent experimental works^{24,57} have shown the possibility to conduct the noise measurement of nanostructures with an ac field. The ability to distinguish the origin of the resonant transport may be helpful to answer the controversy about the origin of the 0.7 anomaly in quantum point contacts, where the zerobias anomaly is due to whether the formation of a strongly correlated Kondo peak⁵⁸ or a single-particle resonant peak^{59,60} is still under debating. We expect further studies on the fingerprint of the correlation effects in the noise behavior may be carried out in the near future to reveal the underlying physics of the 0.7 anomaly.

IV. CONCLUSIONS

In conclusion, we have presented a Floquet–Green's function formalism to investigate the noise properties of quantum dots in the ac-Kondo regime. In principle, the ac effect can be considered nonperturbatively via the infinite Floquet states. The Coulomb correlation is taken into account by the infinite-*U* NCA. Our numerical results based on the NCA are reliable both in the temperature regimes above and below T_K . However, it fails to reproduce the Fermi-liquid behavior at very low temperature $T \ll T_K$. For Kondo dots operate in the zero-temperature limit, the ac transport has been previously studied with the slave-boson mean-field approximation.³⁸ Our results show that the Kondo peak is robust against a weak ac gate modulation. Significant suppression of the Kondo peak can be observed when the ac gate field becomes strong. The photon-assisted noise of a Kondo dot as a function of the dc voltage does not show singularities which are expected for a noninteracting resonant quantum dot at integer $eV/\hbar\Omega$, where V is the dc bias voltage and Ω is the ac frequency. These results suggest that we can tell the noninteracting resonant transport apart from the Kondo resonance by the photon-assisted noise measurement, which cannot be distinguished via the conductance measurement.

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APPENDIX

In this appendix, we outline the derivation to reach the resolvent form of the Floquet–Green's function. It resembles much to the resolvent form of the Green's function at steady state. The starting point is the definition of the Green's function in the time domain

$$\left[i\hbar\frac{\partial}{\partial t} - H(t)\right]G(t,t') = \delta(t-t').$$
 (A1)

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Take the Fourier transform to the two side of Eq. (A1)

$$\int e^{i\omega(t-t')}dt' \left[i\hbar\frac{\partial}{\partial t} - H(t)\right] G(t,t') = 1.$$
 (A2)

The Fourier transform of the Green's function is defined as

$$G(t,t') = \int \frac{d\omega'}{2\pi} e^{-\omega'(t-t')} G(t,\omega').$$
(A3)

Inserting Eq. (A3) into Eq. (A2), we have

$$\int e^{i\omega(t-t')} dt' \left[i\hbar \frac{\partial}{\partial t} - H(t) \right] \int \frac{d\omega'}{2\pi} e^{-\omega'(t-t')} G(t,\omega')$$

$$= \int \int e^{i(\omega-\omega')(t-t')} \left[\hbar \omega' + i\hbar \frac{\partial}{\partial t} - H(t) \right] G(t,\omega') dt' \frac{d\omega'}{2\pi}$$

$$= \int \delta(\omega - \omega') d\omega' [\hbar \omega' - \mathcal{H}_F(t)] G(t,\omega')$$

$$= [\hbar \omega - \mathcal{H}_F(t)] G(t,\omega) = \mathbf{1}, \qquad (A4)$$

where we have introduced the Floquet-Hamiltonian $\mathcal{H}_{F}(t) = H(t) - i\hbar \frac{\partial}{\partial t}$.

Therefore, we arrive at the resolvent form of the Green's function

$$G(t,\omega) = \frac{1}{\hbar\omega - \mathcal{H}_F(t)}.$$
 (A5)

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